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473. Proposed by FRANK R. MORRIS, Gendale, Calif.

What is the length of the longest rectangle an inch wide that can be placed inside another rectangle 12 inches long and 8 inches wide? Obtain the result correct to the third decimal.

SOLUTION BY WILLIAM W. JOHNSON, Cleveland, Ohio.

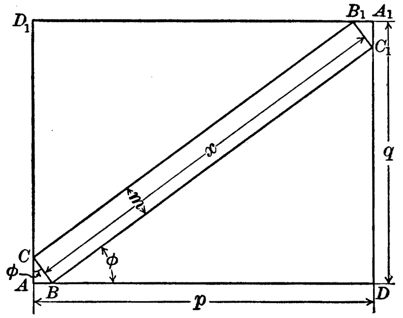
Using the notation of the figure, we have $AB = m \sin \phi$, $BD = x \cos \phi$ and $AB + BD = p$. Then,

$$m \sin \phi + x \cos \phi = p. \tag{1}$$

$AC = m \cos \phi$, $CD_1 = DC_1 = x \sin \phi$ and $AC + CD_1 = q$. Then,

$$x \sin \phi + m \cos \phi = q. \tag{2}$$

Solving (1) and (2) for $\sin \phi$ and $\cos \phi$, we get



$$\sin \phi = \frac{pm - qx}{m^2 - x^2}, \quad \text{and} \quad \cos \phi = \frac{qm - px}{m^2 - x^2}.$$

Squaring, adding, and putting $\sin^2 \phi + \cos^2 \phi = 1$, we get

$$\left[\frac{pm - qx}{m^2 - x^2}\right]^2 + \left[\frac{qm - px}{m^2 - x^2}\right]^2 = 1.$$

Expanding, clearing of fractions and reducing we obtain

$$x^4 - (p^2 + q^2 + 2m^2)x^2 + 4pmqx - (p^2 + q^2 - m^2)m^2 = 0.$$[1]

Putting, $p = 12$, $q = 8$, and $m = 1$, we get

$$x^4 - 210x^2 + 84x - 207 = 0.$$

Solving this equation by Horner's method, we find $x = 13.5176$. Therefore, the length of the inscribed rectangle is 13.5176 inches.

Also solved by H. C. FEEMSTER and J. A. CAPARO.

CALCULUS.

390. Proposed by WILSON L. MISER, University of Arkansas.

Show that the triangle whose area is a constant and whose perimeter is a minimum is equilateral.

SOLUTION BY J. A. CAPARO, University of Notre Dame.

Let ABC be the given triangle of constant area equal to k. Denote the base AB of the triangle by x, the perpendicular CD upon AB by y, the perimeter by P and the angle CAB by α.

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[1] See Merriman's *The Solution of Equations*, p. 20, 4th ed.

Then,

$$k = \frac{xy}{2}. \qquad (1)$$

Now, $P = AC + CB + AB$, and for a minimum $dP/dy = 0$ and $\delta P/\delta\alpha = 0$. But, $AC = y/\sin\alpha = y\csc\alpha$; and since $\overline{CB}^2 = \overline{CD}^2 + \overline{DB}^2$ and $DB = AB - AD = x - y\cot\alpha$,

$$CB = [y^2 + (x - y\cot\alpha)^2]^{\frac{1}{2}} = \left(y^2\csc^2\alpha + \frac{4k^2}{y^2} - 4k\cot\alpha\right)^{\frac{1}{2}}.$$

Then

$$P = y\csc\alpha + \left(y^2\csc^2\alpha + \frac{4k^2}{y^2} - 4k\cot\alpha\right)^{\frac{1}{2}} + \frac{2k}{y}.$$

$$\therefore \quad \frac{dP}{dy} = \csc\alpha + \frac{1}{2}\left(y^2\csc^2\alpha + \frac{4k^2}{y^2} - 4k\cot\alpha\right)^{-\frac{1}{2}}\left(2y\csc^2\alpha - \frac{8k^2}{y^3}\right) - \frac{2k}{y^2} = 0,$$

which can be written

$$(y^2\csc\alpha - 2k)[(y^4\csc^2\alpha + 4k^2 - 4ky^2\cot\alpha)[^{\frac{1}{2}} - (2k + y^2\csc\alpha) = 0.$$

Hence,

$$y^2\csc\alpha - 2k = 0 \qquad (2)$$

and

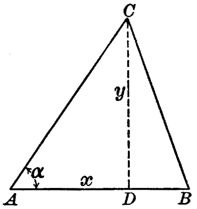
$$y^4\csc^2\alpha + 4k^2 - 4ky^2\cot\alpha = (2k + y^2\csc\alpha)^2,$$

which reduces to

$$-4ky^2\cot\alpha = 4ky^2\csc\alpha.$$

Hence,

$$-\cot\alpha = \csc\alpha \quad \text{or} \quad \alpha = 180^\circ.$$



Substituting (1) in (2) we have $y^2\csc\alpha = xy$ or $\sin\alpha = y/x$; and since $\sin\alpha = y/AC$, $AC = x$, that is, $AC = AB$. Also

$$dP/d\alpha = -y\csc\alpha\cot\alpha + \tfrac{1}{2}(y^2\csc\alpha + 4k^2/y^2 - 4k\cot\alpha)^{-\frac{1}{2}}(-2y^2\csc^2\alpha\cot\alpha + 4k\csc^2\alpha) = 0;$$

or reducing,

$$\cos\alpha\cdot(y^4\csc^2\alpha + 4k^2 - 4ky^2\cot\alpha)^{\frac{1}{2}} = 2k - y^2\cot\alpha.$$

Squaring both sides and reducing, we get

$$(y^2\cot\alpha - k)(1 - \cos^2\alpha) = 0.$$

Hence,

$$1 - \cos^2\alpha = 0 \quad \text{and} \quad y^2\cot\alpha - k = 0. \qquad (3)$$

Solving (2) and (3) for α, we get $\tan\alpha = 2\sin\alpha$. Hence $\alpha = 60^\circ$; and since we have shown above that $AB = AC$ and $\alpha = 60^\circ$ is the included angle between these two lines, it follows that $CB = AB = AC$.

Also solved by FRANK IRWIN.

391. Proposed by H. B. PHILLIPS, Massachusetts Institute of Technology.

If $0 < \lambda < 1$ and $0 < x < \pi$ show that the function $(\sin\lambda x)/(\sin x)$ increases as x increases.

I. SOLUTION BY H. S. UHLER, Yale University.

Applying the well-known formula

$$\sin\theta = \theta\left[1 - \left(\frac{\theta}{\pi}\right)^2\right]\left[1 - \left(\frac{\theta}{2\pi}\right)^2\right]\left[1 - \left(\frac{\theta}{3\pi}\right)^2\right]\cdots, \qquad [\theta^2 < \infty],$$